

A Quadratic Programming  
Analysis of Grain Marketing in Ohio

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ABSTRACT

A constrained two period multi-product optimization model is developed to determine equilibrium corn, soybean and wheat prices in three export markets and the flow of grain from five origins in Ohio to export and domestic demand centers. The MINOS technique can solve this non-linear programming problem in an effective and efficient way.

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## A Quadratic Programming Analysis of Grain Marketing in Ohio

### Introduction

Numerous regional and national grain marketing and transportation studies were completed within the last decade. Cost minimization methods were frequently employed to analyze the total transportation and processing cost associated with the movement of grain from supply centers through the marketing system to final demand points [Baldwin and Larson, Koo, Thompson and Larson, Fedeler, Heady and Koo, Tyrchniewicz and Tosterud, and Sorenson and Fuller]. In these cost minimization models, demand and supply coefficients were exogenously set for each region and all demand and supply functions were assumed to be perfectly inelastic.

Marketing and transportation models, which maximize profits, were also developed [Ladd and Lifferth, Baumel, Miller and Drinka and Solomon]. In these models, all prices were exogenously determined; therefore, the demand functions at final destinations were made perfectly elastic. To offset this limiting assumption, the authors frequently constrained production at supply centers, constrained transportation and storage capacities, and/or limited shipments of grain from origins to destinations.

Takayama and Judge have argued that "the assumptions of fixed regional final product prices and unlimited (perfectly elastic) regional demand (and supply) may be tenuous ones. The assumptions become even more questionable when they are couched within the framework of an industry, since the level of output can affect the level and structure of prices." Inaccurate price levels then identify erroneous shipment

patterns from supply centers to demand points as these models minimize net cost or maximize net profit for shipping grain.

To incorporate elastic and/or inelastic demand and supply functions into programming models, it has been argued that quadratic programming techniques should be substituted for linear programming algorithms [House, and McCarl and Spreen]. Unfortunately, lack of powerful and efficient quadratic programming computer techniques prevented large scale models from being formulated. Duloy and Norton introduced a grid linearization procedure which required segmenting the respective demand and supply functions into segments and specifying a convex combination constraint. Since the Duloy-Norton grid linearization technique is complex and requires considerable CPU time, the process has not been used to solve regional or national grain marketing models.

Recently, a technique called MINOS was introduced to solve nonlinear programming problems [Murtagh and Saunders]. Relative to the Duloy-Norton technique, the MINOS procedure is relatively simplistic and CPU time is reduced. The purpose of this paper is to demonstrate the use of the MINOS technique to determine the optimum shipments of grain from origins in Ohio to export and domestic demand points. Net farm income is maximized for grain producers and the MINOS technique is verified by comparing known shipment patterns to export points and domestic demand centers and export price schedules for the 1976-1981 period with comparable data from the model.

### Methodology

A constrained two period multi-product optimization model is developed to determine optimal equilibrium grain prices in three export

markets and the flow of grain from five Crop Reporting Districts in Ohio to export and domestic demand centers (Figure 1).<sup>1/</sup> The model maximizes a concave quadratic objective function, subject to a set of linear constraints. These constraints describe the storage, transshipment, marketing and transportation activities of country elevators, terminal elevators, river elevators and export elevators. Transshipments of grain among firms and from grain origins in Ohio to final demand points occur by four modes of transportation, truck, rail, barge and combinations of truck, rail and barge movements. A central collection point was designated for each Crop Reporting District in Ohio [McLean].

The objective function consists of linear price-dependent export demand functions and the exogenously determined domestic demand functions; minus total costs for drying, storing and transporting grain. Domestic demands are fixed because these demands are small relative to the volume of grain shipped to export markets. The export demand equations are of the form  $P = a - bQ$ . Values for the coefficients  $a$  and  $b$  were unavailable in the literature and were therefore estimated using elasticities reported by Ray and Richardson. For the derivation of the export demand functions, the concept of proportional demand functions was adopted. Proportional demand functions assume that a region's proportion or share of national demand is equal to its proportion or share of national production. This assumption implies in economic terms that the region faces the same elasticity of demand as the whole nation [Meister, Chowdhury, Ott, and Netayarakas].

The objective function is maximized with a nonlinear system optimizer, MINOS [Murtaugh and Saunders]. For this technique, the user

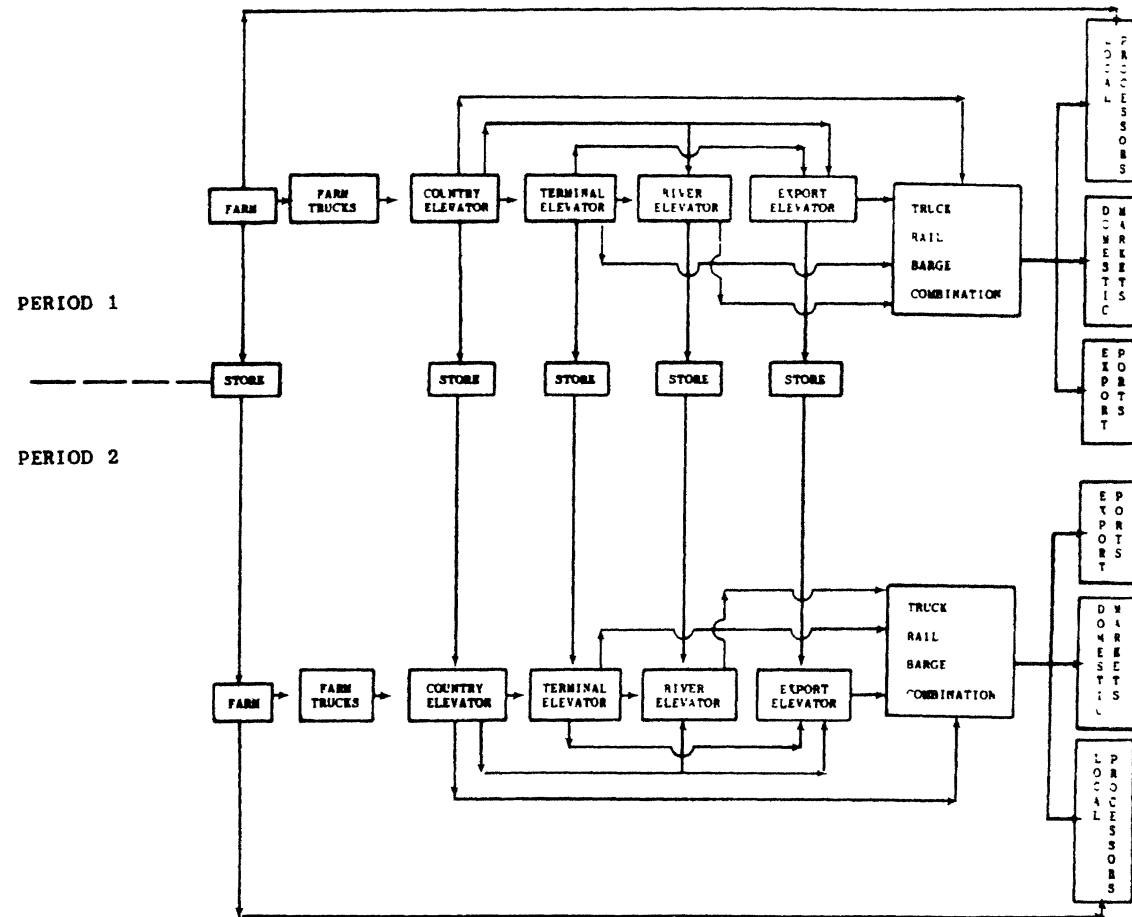


Figure 1: A Schematic Diagram Showing the Flow of Grain from Farms to Final Destinations

provides a set of FORTRAN statements defining the objective function and its gradients along with the partial derivations of the objective function with respect to each of the variables activity level. In this study, these were the slopes of the demand functions. That is, the slopes of the demand function are multiplied by the equilibrium export quantities and the product subtracted from the intercepts to generate the endogenous export prices of the form  $P = a - bQ$ .

The Kuhn-Tucker conditions for a unique optimal solution are satisfied since the objective function is concave and the constraints are convex. The optimum solution to this mathematical formulation provides the set of equilibrium export prices, quantities demanded and grain flows among regions and firms. The equilibrium prices times the equilibrium quantities minus the total costs of drying, storing and transporting grain determines the net farm income.

The data used to solve the model were obtained from secondary sources and are reported in an unpublished Ph.D. dissertation [McLean]. Regional harvest, storage and transshipment activities were derived from a 1977 grain flow survey [Hennen, et al]. Estimates of grain production were collected from the Ohio Agricultural Statistics publications. The price series data were compiled from many sources [Coffman, Latis, Grain Marketing News Weekly, and private grain firms in Ohio]. Grain drying and storage costs were obtained from Schwartz and storage capacity by crop reporting district was obtained from the ASCS 1978 grain storage survey. Transportation costs for truck, rail and barge services were obtained from an unpublished source [Free and Stone].

## Results

The base model describes grain production and marketing activities for the annual average level from 1976 to 1981 using a marketing year with two time periods of July to December and January to June. Whenever possible, the base model results are compared to the actual results. In doing the comparison, however, it is recognized that normative models are not designed to duplicate existing situations, but rather to indicate how a given set of resources can be organized to achieve specific objectives.

### Demand for Corn by the Domestic and Export Markets in Period One

The optimal shipments of corn to the domestic and export markets are compared with the actual five-year shipments in Table 1. A total of 176 million bushels of corn were shipped from Ohio to out-of-state destinations in the first period, this exceeded the actual volume shipped by about 12 million bushels. In this period, 155.6 million bushels of corn are shipped by elevators to export ports located in Toledo, East Coast and the Gulf. Toledo records the largest shipments, accounting for 58 percent of the total shipments. Twenty-nine percent is shipped to the East Coast, with the remaining 13 percent going to the Gulf.

The endogenously determined per bushel export prices for corn at the three export locations are: Toledo \$2.53; East Coast \$2.63 and Gulf \$2.70. These equilibrium prices are lower than the five-year averages used to estimate the export demand functions because within the optimal solution corn exports exceed actual exports by about 12 million bushels.



Table 1: Optimal Base Solution and the Five Year Average Receipts of Corn, Soybeans and Wheat by the Domestic and Export Markets from Ohio Shippers, 1976-1981

Commodity	Time Period <sup>a/</sup>	Toledo		East Coast		Gulf		New York		Pennsylvania		North Carolina		South Carolina		Total	
		Base	5-Year	Base	5-Year	Base	5-Year	Base	5-Year	Base	5-Year	Base	5-Year	Base	5-Year	Base	5-Year
		Model	Average	Model	Average	Model	Average	Model	Average	Model	Average	Model	Average	Model	Average	Model	Average
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Corn	1	89.5	83.7	45.6	41.2	20.4	18.5	4.6	4.6	6.0	6.0	5.8	5.8	4.1	4.1	176.0	163.9
	2	27.2	27.2	37.2	35.0	16.9	16.2	6.7	6.7	4.7	4.7	6.0	6.0	5.3	5.3	104.0	101.1
Sub-total		116.7	110.9	82.8	76.2	37.3	34.7	11.3	11.3	10.7	10.7	11.8	11.8	9.4	9.4	280.0	265.0
Export Prices	1	2.53	2.94	2.63	3.35	2.70	3.42	-	-	-	-	-	-	-	-	-	-
\$/bu.	2	2.95	2.96	2.97	3.42	3.16	3.44	-	-	-	-	-	-	-	-	-	-
Soybeans	1	36.3	36.9	4.6	4.5	7.0	7.0	-	-	-	-	5.9	5.9	5.1	5.1	58.9	59.4
	2	15.8	14.8	8.7	8.0	4.7	4.5	-	-	-	-	2.0	2.0	1.0	1.0	32.2	30.3
Sub-total		52.1	51.7	13.3	12.5	11.7	11.5	-	-	-	-	7.9	7.9	6.1	6.1	91.1	89.7
Export Prices	1	8.38	8.14	8.49	8.56	8.62	8.53	-	-	-	-	-	-	-	-	-	-
\$/bu.	2	6.90	8.12	7.09	8.58	7.53	8.57	-	-	-	-	-	-	-	-	-	-
Wheat	1	14.2	12.7	7.0	6.2	3.0	2.6	2.5	2.5	6.9	6.9	-	-	-	-	33.6	30.9
	2	3.6	0.3	4.3	3.3	0.2	0.1	2.5	2.5	3.0	3.0	-	-	-	-	13.6	9.2
Sub-total		17.8	13.0	11.3	9.5	3.2	2.7	5.0	5.0	9.9	9.9	-	-	-	-	47.2	40.1
Export Prices	1	3.07	4.06	3.11	4.41	3.20	4.46	-	-	-	-	-	-	-	-	-	-
\$/bu	2	-84.4	3.95	1.76	4.26	-3.20	4.36	-	-	-	-	-	-	-	-	-	-
Total		186.6	175.6	107.4	98.2	52.2	48.9	16.3	16.3	20.6	20.6	19.7	19.7	15.5	15.5	418.3	394.8

<sup>a/</sup> Time period one is July to December and time period two is January to June.

Source: McLean

The demands by the out-of-state domestic markets, that is, New York, Pennsylvania, North and South Carolina, were determined exogenously. These demands equal slightly more than 20 million bushels in period one. The demand by the local market is synonymous to the amount of corn demanded by five types of processing firms (feed grinders, feed manufacturers, a corn processor, soybean processors and corn millers) within Ohio. These local demands equalled 30.9 million bushels.

#### Demand for Soybeans by the Domestic and Export Markets in Period One

Optimal shipments of 58.9 million bushels of soybeans flowed from Ohio to export ports and domestic markets within the United States in the first time period. This is only 500,000 bushels less than the actual five-year average shipments (Table 1). Exports account for 47.9 million bushels or 81 percent of total shipments. Optimal shipments to each port closely approximate actual shipments for the five-year average. Therefore, equilibrium prices by port nearly equaled the five-year average prices. North and South Carolina each receive about 6 and 5 million bushels, respectively. The local Ohio demand equaled 16.2 million bushels in the optimal solution.

#### Demand for Wheat by the Domestic and Export Markets in Period One

A total of 33.6 million bushels of wheat was shipped from Ohio to domestic and export markets during period one; this is 2.7 million more than the five-year average (Table 1). The endogenously determined flows to the export ports are 24.2 million bushels of wheat. Shipments of 14.2 million bushels are made through the Toledo port, 7 million bushels flow to the East Coast while the Gulf records receipts of 3 million

bushels. The optimal shipments are higher than the five-year average and the equilibrium export prices are lower than the five-year averages.

With respect to the domestic demand for wheat, New York and Pennsylvania receive about 2.5 million bushels and 6.9 million bushels, respectively. The local Ohio demand equals 17.4 million bushels.

Demand for Corn, Soybeans and Wheat by the Domestic and Export Markets in Period Two

In period two, the optimal outflows of corn are approximately 104 million bushels which is slightly larger than the five-year average outflow (Table 1). Toledo loses its dominance to the East Coast because during the first three months the Great Lakes are frozen. Whenever shipments through the Great Lakes are impossible, more grain is shipped to the East Coast and the Gulf. The per bushel export prices are: Toledo \$2.95, East Coast \$2.98 and Gulf \$3.44. These prices are higher than in the first period, a consistent finding, given that prices are usually higher in the non-harvesting periods than at harvest time. The equilibrium East Coast and Gulf prices are lower than the five-year average prices because optimal corn shipments are higher.

The domestic markets receive a total of 22.7 million bushels of corn. Optimal corn shipments to the local Ohio market equal 24.9 million bushels.

Thirty-two million bushels of soybeans are shipped to out-of-state destinations in the second time period. This is only 1.9 million bushels more than the five-year average (Table 1). Exports continued to be the major outflow, accounting for 29.3 million bushels. More soybeans flowed to the East Coast under the base solution than in the previous period. Shipments to the other two export ports fell by more

than half in the case of Toledo and less than one-third for the Gulf. There is clearly a smaller supply available during the second period, thus explaining the short fall in shipments to the Gulf. At the same time it should be recognized that, during the winter months there is some reallocation of shipments between Toledo and the East Coast ports. The equilibrium export prices in period two are lower than the five-year average because of the higher level of shipments to these ports in the optimal solution. North and South Carolina receive a total of three million bushels of soybeans in period two. The local Ohio demand equals 6.7 million bushels.

Optimal flows of wheat are much higher in the base model than the five-year averages. Due to the large disparities between the amounts shipped by farms to export elevators and country elevators in the base model and the five-year average solution, the endogenously determined prices at Toledo and the Gulf are negative. The excess supply is generated when the assumption was made that corn is the only grain consumed by livestock on farms. Therefore, all the wheat produced is allowed to flow through the marketing system. In an economic sense, no supplier would continue to make shipments to the point where prices become negative. It is, therefore, obvious that some of this excess supply would be stored as carry out in period two.

#### Net Farm Income

Having discussed the flows of corn, soybeans and wheat, levels of export prices and demands, the net farm incomes of producers from the sale of these commodities can now be derived. Net farm income for the two periods of the model equal \$1,893 million. The average five-year

cash receipts for marketing corn, wheat and soybeans were estimated to be \$1,745 million; this is only slightly below the base model results.

#### Conclusions and Implications

The MINOS technique to determine optimum shipments of grain from origins in Ohio to export and domestic demand points represents an effective and efficient method to use in modelling grain marketing and transportation systems. The perfectly elastic and/or inelastic regional demand and supply assumptions associated with the traditional linear programming models are relaxed by the MINOS technique. Although space is not available to report the results in this paper, the model can also be used to evaluate the economic effects of changes in demand and supply functions and transportation and storage conditions on the net incomes of farmers.

The estimated export demand functions for the base model were able to predict with reasonable levels of accuracy the Ohio grain flows, quantities demanded and prices for two time periods for the five year average 1976-1981. The export and domestic market prices minus transportation and marketing costs generated optimal five year average net farm income of \$1.9 billion which was only 8.5% higher than actual net farm income from the sale of corn, soybeans and wheat in Ohio during the 1976-1981 period. The predictions for quantities demanded, grain flows, prices and net farm incomes can be further refined and improved with three modifications: (1) grain is stored or carry-out occurs at the end of the second time period, (2) wheat is fed to livestock in Ohio, and (3) permit interactions between export demands and domestic demands. Such changes will likely increase the predicted export prices,

increase the quantity of grain in storage and decrease the quantity of grain flowing to export points in the model. This will further improve price projections and export flows relative to the actual prices and flows for the 1976-1981 period. The increase in storage cost and the decrease in the quantity demanded at export points will reduce projected net farm income to more nearly equal actual levels.

With these modifications, the impact of grain production, transportation, marketing and export policies on grain flows, prices and net farm incomes can be more accurately predicted using the MINOS technique for national, regional or state models. For example, any policy to increase export demand will increase prices and result in further expansions in grain supply. Since large grain terminals, train loading stations and river houses primarily ship export grain, such a policy would alter the structure of the grain industry. Because most grain shipments to the East Coast move via train load rates, the structure of the transportation system would also be altered.

If the production increases of the 1970s were repeated in the 1980s, then price levels and net farm income could fall significantly, unless matched by increases in export demand. If in the 1980s, the value of the dollar decreases, world economic conditions improve and export demand increases, export prices and net farm income would increase causing more grain to flow to export points and less grain to flow to the domestic inter-state market and the livestock industry within Ohio.

#### Footnotes

1/ In this manuscript, grain includes corn, soybeans and wheat. The three export markets are Toledo, East Coast and the Gulf. Domestic demand points include five grain processors in Ohio and grain deficit markets located in New York, Pennsylvania, North Carolina and South Carolina.

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